



MATHEMATICS

MAXIMUM MARKS: 80

TIME ALLOWED: THREE HOURS

(CANDIDATES ARE ALLOWED ADDITIONAL 15 MINUTES FOR ONLY READING THE PAPER. THEY MUST NOT START WRITING DURING THIS TIME.)

- *This Question Paper consists of three sections A, B and C.*
- *Candidates are required to attempt all questions from Section A and all questions EITHER from Section B OR Section C.*
- *Section A: Internal choice has been provided in two questions of two marks each, two questions of four marks each and two questions of six marks each.*
- *Section B: Internal choice has been provided in one question of two marks and one question of four marks.*
- *Section C: Internal choice has been provided in one question of two marks and one question of four marks.*
- *All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.*
- *The intended marks for questions or parts of questions are given in brackets [].*
- *Mathematical tables and graph papers are provided.*

CLASSES

SECTION A - 65 MARKS

Question 1: In subparts (i) to (xi) choose the correct option, and in subparts (xii) to (xv) answer as instructed, $15 \times 1 = 15$

(i) If A and B are 2×2 matrices and $AB = 0$, then which of the following must be true?

- (a) $A = 0$ or $B = 0$
- (b) $\det(A) = 0$ or $\det(B) = 0$
- (c) A and B are symmetric
- (d) A and B are orthogonal

(ii) The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + y\left(\frac{dy}{dx}\right)^4 = 0$ are:

- (a) 2, 3
- (b) 2, 1

- (c) 3, 2
(d) 2, 4

(iii) If $x^2 + y^2 = 25$, then $\frac{dy}{dx} =$

- (a) $\frac{x}{y}$
(b) $-\frac{x}{y}$
(c) $\frac{y}{x}$
(d) $-\frac{y}{x}$

(iv) **Assertion:** For any square matrix A , $\det(3A) = 3\det(A)$.

Reason: Multiplying a row by 3 multiplies the determinant by 3.

- (a) Both A and R are true, and R is the correct explanation of A.
(b) Both A and R are true, but R is not the correct explanation.
(c) A is true, R is false.
(d) A is false, R is true.

(v) Three distinct numbers are selected from $\{1, 2, 3, 4, 5, 6\}$. The probability that the smallest number is 2 and the largest is 5 is:

- (a) $\frac{1}{10}$
(b) $\frac{1}{15}$
(c) $\frac{1}{20}$
(d) $\frac{1}{30}$

(vi) If $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, then $A^5 =$

- (a) $\begin{bmatrix} 10 & 0 \\ 0 & 15 \end{bmatrix}$
(b) $\begin{bmatrix} 32 & 0 \\ 0 & 243 \end{bmatrix}$
(c) $\begin{bmatrix} 25 & 0 \\ 0 & 35 \end{bmatrix}$
(d) $\begin{bmatrix} 7 & 0 \\ 0 & 8 \end{bmatrix}$

(vii) Statement 1: A differentiable function is always continuous.

Statement 2: $f(x) = |x - 1|$ is differentiable at $x = 1$.

- (a) S1 true, S2 false
(b) S2 true, S1 false
(c) Both true
(d) Both false

(viii) If $f(x) = 3x^3 - 5x^2 + 7$ and $f'(1) = 4$, then the value of the coefficient of x^3 is:

- (a) 3



- (b) 4
(c) 5
(d) 6

(ix) Statement 1: A relation R on set $A = \{1,2,3\}$ defined by $R = \{(1,1), (2,2)\}$ is symmetric.
Statement 2: For symmetry, if $(a, b) \in R$ then $(b, a) \in R$.

- (a) S1 true, S2 false
(b) S2 true, S1 false
(c) Both true
(d) Both false

(x) Assertion: $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi$.

Reason: $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$ if $xy < 1$.

- (a) Both A and R true, R explains A
(b) Both true, R does not explain A
(c) A true, R false
(d) A false, R true

(xi) Given $P(A) = 0.5$, $P(B) = 0.4$, and $P(A \cap B) = 0.2$, then $P(A \cup B) =$

- (a) 0.7
(b) 0.9
(c) 0.8
(d) 0.6

(xii) If A is a 2×2 matrix with $\det(A) = 5$, and C is the matrix of cofactors of A , find $\det(C)$. [1]

(xiii) Let $A = \{1,2,3\}$. If a relation R on A is defined by $R = \{(1,2), (2,3), (1,3)\}$, classify R (reflexive/symmetric/transitive). [1]

(xiv) The function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^3$ is one-to-one. Give a reason. [1]

(xv) In a batch of 5 items, 2 are defective. Two items are drawn at random without replacement. What is the probability that both are defective? [1]

Question 2 [2]:

(i) If $\log(x^2 + y^2) = 2 \tan^{-1} \left(\frac{y}{x} \right)$, prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

(OR)

(ii) Find the value(s) of k for which $f(x) = kx^3 - 3x^2 + 6x + 5$ is increasing for all real x .

Question 3 [2]:

Find the point on the curve $y = 3x^2 - 2x$ where the tangent is parallel to the line $y = 4x - 1$.

Question 4 [2]:

Solve: $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

Question 5 [2]:

(i) If $\int e^{2x} \sin(3x) dx = \frac{e^{2x}(A \sin 3x + B \cos 3x)}{C} + D$, find $A + B + C$.
(OR)

(ii) Evaluate $\int_0^{\pi/2} \cos^2 x dx$.

Question 6 [2]:

Let $f(x) = \sin^{-1}(3x - 4x^3)$. Compute $f\left(\frac{1}{2}\right) + f\left(\frac{\sqrt{3}}{2}\right)$.

Question 7 [4]:

Solve: $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}3x$.

Question 8 [4]:

If $y = e^{2x}(A \cos 3x + B \sin 3x)$, prove that $\frac{d^3y}{dx^3} - 4\frac{dy}{dx} + 13y = 0$.

Question 9 [4]:

(i) Solve $\frac{dy}{dx} = \frac{y}{x} + \sin\left(\frac{y}{x}\right)$.

(OR)

(ii) Solve $(x^2 + 1)\frac{dy}{dx} + 2xy = 4x^2$.

Question 10 [4]:

A bag contains 4 red and 6 blue balls. Two balls are drawn at random without replacement.

- Find the probability that both are red.
- Find the probability that one is red and one is blue.
- If the first ball is red, what is the probability that the second is blue?

(OR)

In a college, 60% students are boys. 30% of boys and 40% of girls play cricket. A randomly chosen student plays cricket.

- Define events and write probabilities.
- Find the probability that the student is a girl.
- Interpret the result.

Question 11 [6]:

Three shops A, B, C sell pens, pencils, and erasers. Shop A sold 12 pens, 8 pencils, 10 erasers for ₹600. Shop B sold 5 pens, 10 pencils, 8 erasers for ₹400. Shop C sold 8 pens, 6 pencils, 12 erasers for ₹500.

- Formulate a system of equations.
- Solve using matrix method.
- Find the price of each item.

Question 12 [6]:

(i) $\int \frac{3x+5}{x^2+5x+6} dx$

(OR)

(ii) $\int \frac{x+1}{x^2+4x+5} dx$

Question 13 [6]:

(i) A rectangular box with a square base and open top has volume 32 cm^3 . Find the dimensions for minimum surface area.

(OR)

(ii) A particle moves along a line with position $s(t) = t^3 - 6t^2 + 9t + 2$. Find:

- (a) Time when velocity is zero.
- (b) Acceleration at those times.
- (c) Distance traveled in first 4 seconds.

Question 14 [6]:

The number of emails received per hour follows Poisson distribution with mean 3.

- (i) Find $P(X = 0)$ and $P(X = 4)$.
- (ii) Write probability distribution up to $X = 4$.
- (iii) Find the expected number of emails.
- (iv) If more than 5 emails per hour is considered overload, should they upgrade?

SECTION B - 15 MARKS

Question 15 [5]:

(i) Statement 1: The cross product of two parallel vectors is zero.

Statement 2: The dot product of two perpendicular vectors is zero.

- (a) S1 true, S2 false
- (b) S2 true, S1 false
- (c) Both true
- (d) Both false

(ii) The direction ratios of a line perpendicular to the plane $2x - 3y + 4z = 5$ are:

- (a) $\langle 2, -3, 4 \rangle$
- (b) $\langle 1, 1, 1 \rangle$
- (c) $\langle -2, 3, -4 \rangle$
- (d) $\langle 0, 0, 1 \rangle$

(iii) If $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \times \vec{b} = \vec{0}$, then:

- (a) $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$
- (b) $\vec{a} \perp \vec{b}$
- (c) $\vec{a} \parallel \vec{b}$
- (d) None

(iv) The angle between $\vec{i} + \vec{j} + \vec{k}$ and $\vec{i}$ is:

- (a) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$
- (b) $\frac{\pi}{4}$
- (c) $\frac{\pi}{6}$
- (d) $\frac{\pi}{3}$

(v) Find $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b})$ if $|\vec{a}| = 3$, $|\vec{b}| = 4$, and $\vec{a} \cdot \vec{b} = 5$.

Question 16 [2]:

(i) Let $A(1,2,3)$, $B(4,5,6)$, $C(7,8,9)$. Find the area of triangle ABC.

(OR)

(ii) Find x if vectors $\vec{p} = x\vec{i} + \vec{j} + \vec{k}$ and $\vec{q} = \vec{i} + x\vec{j} + \vec{k}$ are perpendicular.

Question 17 [4]:

(i) Find the equation of the plane through $(1,2,3)$ and perpendicular to the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$.

(OR)

(ii) Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$.

Question 18 [4]: Find the area bounded by $y = x^2$, x -axis, and lines $x = 1$, $x = 3$.

SECTION C - 15 MARKS

Question 19 [5]:

(i) The average cost function $AC = 2x + 5 + \frac{12}{x}$ is minimum at $x =$

- (a) 2
- (b) 3
- (c) 4
- (d) 5

(ii) If the demand function is $p = 10 - x$, then marginal revenue at $x = 3$ is:

- (a) 4
- (b) 5
- (c) 6
- (d) 7

(iii) The regression line $y = a + bx$ has $b = 0$ if:

- (a) $r = 0$
- (b) $r = 1$
- (c) $r = -1$
- (d) None

(iv) If marginal cost $MC = 3x^2 - 6x + 4$ and fixed cost is 100, find total cost.

(v) Find the line of regression of y on x if $\Sigma x = 30$, $\Sigma y = 40$, $\Sigma xy = 200$, $\Sigma x^2 = 150$, $n = 5$.

Question 20 [4]:

(i) The total cost function $C(x) = x^3 - 6x^2 + 13x + 15$. Find:

- (a) Marginal cost
- (b) Output for minimum average cost

(OR)

(ii) A firm has fixed cost ₹500 and variable cost per unit ₹10. If the demand function is $p = 50 - 2x$, find the break-even points.

Question 21 [4]:

For the data:

X	1	2	3	4	5
Y	2	4	5	8	11

Find the regression line of Y on X and estimate Y when $X = 6$.

Question 22 [4]:

(i) Solve the LPP: Maximize $Z = 3x + 4y$ subject to $x + y \leq 5$, $2x + y \leq 8$, $x, y \geq 0$.

(OR)

(ii) A dietitian wants to minimize cost with two foods F1 and F2. F1 costs ₹4 per unit and F2 costs ₹6 per unit. F1 provides 3 units of protein and 4 units of carbs; F2 provides 4 units of protein and 2 units of carbs. Minimum requirements: 12 units protein, 12 units carbs. Formulate the LPP.